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GENERALIZED FAMILY OF EFFICIENT ESTIMATORS OF POPULATION MEDIAN USING INFORMATION ON TWO AUXILIARY VARIABLES

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ABSTRACT

Using prior information of population parameters based upon two auxiliary variables, a generalized family of efficient estimators under two-phase simple random sampling design has been proposed for estimating the population median of variable under study. The expressions of gain in efficiencies of proposed family of estimators with respect to mean squared error and bias over the existing ones have been obtained, which shows that better estimators can be obtained from the proposed family under the given practical situations. Numerical and graphical illustrations of the results obtained have also been made by taking some empirical populations considered in the literature.

KEYWORDS: Auxiliary Variable, Bias, Mean Square Error, Median, Two-Phase Sampling

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Original Article

1. INTRODUCTION

It is well known that median is regarded as the best measure of location of population for the case of skewed population. In survey sampling, some authors such as Gross(1980), Kuk and Mak (1989), Singh et al (2001,2006), Jhajj et al (2013) etc. have defined the estimators of population median based upon variable under study only and using the known information of auxiliary variables highly correlated with variable under study in form of their parameters. In the absence of known information on parameters of auxiliary variables, two-phase sampling design have been widely used for developing efficient estimators of population parameters of variable under study practically and theoretically.

Assuming that information on population median M_Z of auxiliary variable Z (say) is known, but on population median M_X of auxiliary variable X (say) is not known in a trivariate population (X, Y, Z) which is estimated on the basis of first phase sample under two phase sampling design. The variable under study Y (say) may be directly highly correlated with all auxiliary variables or may not be highly correlated with some auxiliary variables but correlated through other auxiliary variables.

Considering a finite population U= (1, 2,… i ,… N) in which Y_i and X_i, Z_i be the values of study variable Y and auxiliary variables X , Z on the i^{th} unit of the population respectively. Let M_Y , M_X and M_Z be the population medians of variables Y and X , Z respectively. Under two- phase sampling design, first phase

sample of size n is selected and variables are observed on sample units. Then second phase simple random sample of size m is drawn from the first phase sample and observations on the variables are noted on selected units. Let $\hat{M}_Y$, $\hat{M}_X$ and $\hat{M}_Z$ denote the sample medians of respective variables Y, X and Z in the second phase sample and corresponding sample medians $\hat{M}'_Y$, $\hat{M}'_X$ and $\hat{M}'_Z$ in first phase sample.

Suppose the values of variable Y on the sample of m units arranged in ascending order are $y_{(1)}, y_{(2)}, \ldots, y_{(m)}$ such that $y_{(t)} \le M_Y \le y_{(t+1)}$ for some integral value of t. Let $p = t/m$ be the proportion of Y values in the sample that are less than or equal to the value of population median M_Y (unknown). If $\hat{p}$ is an estimator of p, the sample median $\hat{M}_Y$ can be expressed as quantiles $\hat{Q}_Y(\hat{p})$ with $\hat{p} = 0.5$.

Defining

$$\rho_{xy} = \rho_{(\hat{M}_X, \hat{M}_Y)} = 4\{P_{11}(x,y) - 1\}, \text{ with } P_{11}(x,y) = P(X \le M_X \cap Y \le M_Y)$$

$$\rho_{yz} = \rho_{(\hat{M}_Y, \hat{M}_Z)} = 4\{P_{11}(y,z) - 1\}, \text{ with } P_{11}(y,z) = P(Y \le M_Y \cap Z \le M_Z)$$

$$\rho_{xz} = \rho_{(\hat{M}_X, \hat{M}_Z)} = 4\{P_{11}(x,z) - 1\}, \text{ with } P_{11}(x,z) = P(X \le M_X \cap Z \le M_Z)$$

Here ρ_{xy}, ρ_{yz} and ρ_{xz} denote the correlation coefficient between estimators $\hat{M}_X$ & $\hat{M}_Y$; $\hat{M}_Y$ & $\hat{M}_Z$ and $\hat{M}_X$ & $\hat{M}_Z$ respectively.

In (2006), Singh et al defined a ratio type estimator of population median M_Y under two phase simple random sampling design using the known value of M_Z in the trivariate population (X, Y, Z) as

$$\hat{M}_S = \hat{M}_Y \left(\frac{\hat{M}'_X}{\hat{M}_X}\right)^{\alpha_1} \left(\frac{M_Z}{\hat{M}'_Z}\right)^{\alpha_2} \left(\frac{M_Z}{\hat{M}_Z}\right)^{\alpha_3} \tag{1.1}$$

Where $\alpha_i\,(i = 1,2,3)$ are constants.

Up to first order of approximation, they minimized mean squared error of estimator $MSE(\hat{M}_S)$ for:

$$\alpha_1 = \frac{M_X f(M_X)}{M_Y f(M_Y)} \left(\frac{\rho_{xz}\rho_{yz} - \rho_{xy}}{\rho_{xz}^2 - 1}\right),$$

$$\alpha_2 = \frac{M_Z f\left(M_Z\right)\rho_{xz}}{M_Y f\left(M_Y\right)}\left(\frac{\rho_{xz}\rho_{yz}-\rho_{xy}}{\rho_{xz}^2-1}\right)$$

$$\alpha_3 = \frac{M_Z f\left(M_Z\right)}{M_Y f\left(M_Y\right)}\left(\frac{\rho_{xz}\rho_{xy}-\rho_{yz}}{\rho_{xz}^2-1}\right)$$

and obtained the Bias and MSE of optimum estimator of $\hat{M}_S$, given as under:

$$MSE\left(\hat{M}_S\right)_{\min} = \frac{1}{4\left\{f(M_Y)\right\}^2}\left[\left(\frac{1}{m}-\frac{1}{N}\right)-\left(\frac{1}{n}-\frac{1}{N}\right)\rho_{yz}^2-\left(\frac{1}{m}-\frac{1}{n}\right)R_{y.xz}^2\right] \tag{1.2}$$

$$Bias(\hat{M}_S) \cong \frac{M_Y}{8\left\{M_Y f(M_Y)\right\}^2(1-\rho_{xz}^2)^2}$$

$$\times\left[\left(\frac{1}{m}-\frac{1}{n}\right)\left\{\left(\rho_{xy}-\rho_{yz}\rho_{xz}\right)^2-2\rho_{xy}\left(\rho_{xy}-\rho_{yz}\rho_{xz}\right)\left(1-\rho_{xz}^2\right)\right.\right.$$

$$\left.+\frac{M_Y f\left(M_Y\right)}{M_X f\left(M_X\right)}\left(\rho_{xy}-\rho_{yz}\rho_{xz}\right)\left(1-\rho_{xz}^2\right)\right\}$$

$$+\left(\frac{1}{m}-\frac{1}{N}\right)\left\{\left(\rho_{yz}-\rho_{xy}\rho_{xz}\right)^2+2\rho_{xz}\left(\rho_{xy}-\rho_{yz}\rho_{xz}\right)\left(\rho_{yz}-\rho_{xy}\rho_{xz}\right)\right. \tag{1.3}$$

Where $R_{y.xz}^2 = \dfrac{\rho_{xy}^2+\rho_{yz}^2-2\rho_{xy}\rho_{yz}\rho_{xz}}{1-\rho_{xz}^2}$

Using the knowledge on range R_Z of variable Z in addition to its population median M_Z, Gupta et al (2008) defined the estimator of population median M_Y under the same sampling design considered by Singh et al (2006) as

$$\hat{M}_P = \hat{M}_Y\left(\frac{\hat{M}'_X}{\hat{M}_X}\right)^{\gamma_1}\left(\frac{M_Z+R_Z}{\hat{M}'_Z+R_Z}\right)^{\gamma_2}\left(\frac{M_Z+R_Z}{\hat{M}_Z+R_Z}\right)^{\gamma_3} \tag{1.4}$$

Where $\gamma_i\left(i=1,2,3\right)$ are constants.

Up to the first order of approximation, they minimized MSE of $\hat{M}_P$ for

$$\gamma_1 = \frac{M_X f\left(M_X\right)}{M_Y f\left(M_Y\right)}\left(\frac{\rho_{xz}\rho_{yz}-\rho_{xy}}{\rho_{xz}^2-1}\right), \qquad \gamma_2 = \frac{1}{\left(\dfrac{M_Z}{M_Z+R_Z}\right)}\frac{M_Z f\left(M_Z\right)\rho_{xz}}{M_Y f\left(M_Y\right)}\left(\frac{\rho_{xz}\rho_{yz}-\rho_{xy}}{\rho_{xz}^2-1}\right)$$

$$\gamma_3 = \frac{1}{\left(\frac{M_Z}{M_Z + R_Z}\right)} \frac{M_Z f(M_Z)}{M_Y f(M_Y)} \left(\frac{\rho_{xz}\rho_{xy} - \rho_{yz}}{\rho_{xz}^2 - 1}\right)$$

And obtained the bias and mse of optimum estimator of $\hat{M}_P$, given as under :

$$\begin{aligned} Bias(\hat{M}_P) \cong & \frac{M_Y}{8\{M_Y f(M_Y)\}^2 (1-\rho_{xz}^2)^2} \\ & \times\Bigg[\left(\frac{1}{m}-\frac{1}{n}\right)\Big\{\left(\rho_{xy}-\rho_{yz}\rho_{xz}\right)^2 - 2\rho_{xy}\left(\rho_{xy}-\rho_{yz}\rho_{xz}\right)\left(1-\rho_{xz}^2\right) \\ & \qquad + \frac{M_Y f(M_Y)}{M_X f(M_X)}\left(\rho_{xy}-\rho_{yz}\rho_{xz}\right)\left(1-\rho_{xz}^2\right)\Big\} \\ & + \left(\frac{1}{m}-\frac{1}{N}\right)\Big\{\left(\rho_{yz}-\rho_{xy}\rho_{xz}\right)^2 + 2\rho_{xz}\left(\rho_{xy}-\rho_{yz}\rho_{xz}\right)\left(\rho_{yz}-\rho_{xy}\rho_{xz}\right) \\ & \qquad + \frac{M_Y f(M_Y)}{M_Z f(M_Z)}\left(\frac{M_Z}{M_Z+R_Z}\right)\left(\rho_{yz}-\rho_{xy}\rho_{xz}\right)\left(1-\rho_{xz}{}^2\right)\Big\} \\ & + \left(\frac{1}{n}-\frac{1}{N}\right)\Big\{\rho_{xz}{}^2\left(\rho_{xy}-\rho_{yz}\rho_{xz}\right)^2 - 2\rho_{yz}\rho_{xz}\left(\rho_{xy}-\rho_{yz}\rho_{xz}\right)\left(1-\rho_{xz}^2\right) \\ & \qquad + \frac{M_Y f(M_Y)}{M_Z f(M_Z)}\left(\frac{M_Z}{M_Z+R_Z}\right)\rho_{xz}\left(\rho_{xy}-\rho_{yz}\rho_{xz}\right)\left(1-\rho_{xz}^2\right)\Big\}\Bigg] \end{aligned} \tag{1.5}$$

$$MSE\left(\hat{M}_P\right)_{\min} \cong \frac{1}{4\{f(M_Y)\}^2}\left[\left(\frac{1}{m}-\frac{1}{N}\right) - \left(\frac{1}{n}-\frac{1}{N}\right)\rho_{yz}^2 - \left(\frac{1}{m}-\frac{1}{n}\right)R_{y.xz}^2\right] \tag{1.6}$$

$$= MSE\left(\hat{M}_S\right)_{\min}$$

Recently Jhajj et al (2014) defined an efficient family of estimators of median using the same information used by Singh et al (2006) under same sampling design as

$$\hat{M}_{YH} = \hat{M}_Y \left(\frac{M_Z}{\hat{M}'_Z}\right)^{\upsilon_1} \left(\frac{M_Z}{\hat{M}_Z}\right)^{\upsilon_2} e^{\left(\frac{\upsilon_3\left(\hat{M}'_X - \hat{M}_X\right)}{\hat{M}'_X + \hat{M}_X}\right)} \tag{1.7}$$

Here υ_1, υ_2 and υ_3 are constants.

Up to the first order of approximation, they minimized MSE of $\hat{M}_{YH}$ for

$$\upsilon_1 = \frac{M_Z f(M_Z)\rho_{xz}}{M_Y f(M_Y)}\left(\frac{\rho_{xz}\rho_{yz} - \rho_{xy}}{\rho_{xz}^2 - 1}\right), \quad \upsilon_2 = \frac{M_Z f(M_Z)}{M_Y f(M_Y)}\left(\frac{\rho_{xy}\rho_{xz} - \rho_{yz}}{\rho_{xz}^2 - 1}\right)$$

$$\upsilon_3 = \frac{2M_X f(M_X)}{M_Y f(M_Y)}\left(\frac{\rho_{yz}\rho_{xz}-\rho_{xy}}{\rho_{xz}^2-1}\right)$$

And obtained the bias and MSE of optimum estimator of $\hat{M}_{YH}$, given as under:

$$\begin{aligned}
Bias(\hat{M}_{YH}) \cong & \frac{M_Y}{8\{M_Y f(M_Y)\}^2(1-\rho_{xz}^2)^2} \\
& \times\Bigg[\left(\frac{1}{m}-\frac{1}{n}\right)\Big\{\left(\rho_{xy}-\rho_{yz}\rho_{xz}\right)^2-2\rho_{xy}\left(\rho_{xy}-\rho_{yz}\rho_{xz}\right)\left(1-\rho_{xz}^2\right) \\
& \qquad +\frac{M_Y f(M_Y)}{M_X f(M_X)}\left(\rho_{xy}-\rho_{yz}\rho_{xz}\right)\left(1-\rho_{xz}^2\right)\Big\} \\
& +\left(\frac{1}{m}-\frac{1}{N}\right)\Big\{\left(\rho_{yz}-\rho_{xy}\rho_{xz}\right)^2+2\rho_{xz}\left(\rho_{xy}-\rho_{yz}\rho_{xz}\right)\left(\rho_{yz}-\rho_{xy}\rho_{xz}\right) \\
& \qquad -2\rho_{yz}\left(\rho_{yz}-\rho_{xy}\rho_{xz}\right)\left(1-\rho_{xz}^2\right)+\frac{M_Y f(M_Y)}{M_Z f(M_Z)}\left(\rho_{yz}-\rho_{xy}\rho_{xz}\right)\left(1-\rho_{xz}{}^2\right)\Big\} \\
& +\left(\frac{1}{n}-\frac{1}{N}\right)\Big\{\rho_{xz}{}^2\left(\rho_{xy}-\rho_{yz}\rho_{xz}\right)^2-2\rho_{yz}\rho_{xz}\left(\rho_{xy}-\rho_{yz}\rho_{xz}\right)\left(1-\rho_{xz}^2\right) \\
& \qquad +\frac{M_Y f(M_Y)}{M_Z f(M_Z)}\rho_{xz}\left(\rho_{xy}-\rho_{yz}\rho_{xz}\right)\left(1-\rho_{xz}^2\right)\Big\}\Bigg]
\end{aligned} \tag{1.8}$$

$$MSE\left(\hat{M}_{YH}\right)_{\min} \cong \frac{1}{4\{f(M_Y)\}^2}\left[\left(\frac{1}{m}-\frac{1}{N}\right)-\left(\frac{1}{n}-\frac{1}{N}\right)\rho_{yz}^2-\left(\frac{1}{m}-\frac{1}{n}\right)R_{y.xz}^2\right] \tag{1.9}$$

$= MSE\left(\hat{M}_S\right)_{\min}$

2. PROPOSED ESTIMATOR AND ITS RESULTS

Suppose that variable X is highly correlated with variables Y and Z in the trivariate distribution of variables Y, X and Z, but variables Y and Z are correlated to each other only because of their correlation with X. When population median M_Z of variable Z is only known, we propose a new generalized family of estimators of population median M_Y under two phase sampling design described in Section 1 as

$$\hat{M}_{YH\theta} = \left[\hat{M}_Y+\theta\left(\hat{M}'_Y-\hat{M}_Y\right)\right]\left[\exp\left(\frac{2\eta_1\left(\hat{M}_X-\hat{M}'_X\right)}{\left(\hat{M}_X+\hat{M}'_X\right)}\right)\right]\left[\frac{\hat{M}_Z}{\hat{M}'_Z}\right]^{\eta_2}\left[\frac{M_Z}{\hat{M}'_Z}\right]^{\eta_3} \tag{2.1}$$

Where η_1, η_2, η_3 are constants with $\theta > 0$.

Assuming that mean squared error of the estimator $\hat{M}_{YH\theta}$ exists and finite. So, for finding its mean squared error and bias up to first order of approximation, we expand $\hat{M}_{YH\theta}$ in terms of $\varepsilon' s$. up to their second degree by defining $\varepsilon' s$ as

$$\varepsilon_0 = \frac{\hat{M}_Y}{M_Y} - 1, \quad \varepsilon_1 = \frac{\hat{M}'_Y}{M_Y} - 1, \quad \varepsilon_2 = \frac{\hat{M}_X}{M_X} - 1,$$

$$\varepsilon_3 = \frac{\hat{M}'_X}{M_X} - 1, \quad \varepsilon_4 = \frac{\hat{M}_Z}{M_Z} - 1, \quad \varepsilon_5 = \frac{\hat{M}'_Z}{M_Z} - 1$$

provided $|\varepsilon_i| < 1 \ \forall i$, such that $E(\varepsilon_i) \cong 0$ for $i = 0, 1, 2, 3, 4, 5$

Up to first order of approximation, bias and mean squared error of $\hat{M}_{YH\theta}$ obtained are :

$$Bias(\hat{M}_{YH\theta}) = \frac{M_Y}{4}\Bigg[\left(\frac{1}{n}-\frac{1}{N}\right)\left\{\frac{\eta_3(\eta_3+1)}{2}\left(M_Z f(M_Z)\right)^{-2} - \eta_3\rho_{yz}\frac{\left(M_Z f(M_Z)\right)^{-1}}{\left(M_Y f(M_Y)\right)}\right\}$$
$$+\left(\frac{1}{m}-\frac{1}{n}\right)\Bigg\{\frac{\eta_1(\eta_1-1)}{2}\left(M_X f(M_X)\right)^{-2} + \frac{\eta_2(\eta_2-1)}{2}\left(M_Z f(M_Z)\right)^{-2}$$
$$+(1-\theta)\left(\eta_1\rho_{xy}\frac{\left(M_X f(M_X)\right)^{-1}}{\left(M_Y f(M_Y)\right)} + \eta_2\rho_{yz}\frac{\left(M_Z f(M_Z)\right)^{-1}}{\left(M_Y f(M_Y)\right)}\right) + \eta_1\eta_2\rho_{xz}\frac{\left(M_X f(M_X)\right)^{-1}}{\left(M_Z f(M_Z)\right)}\Bigg\}\Bigg] \tag{2.2}$$

$$MSE\left(\hat{M}_{YH\theta}\right) = \frac{M_Y^2}{4}\Bigg[\left(\frac{1}{m}-\frac{1}{N}\right)\left(M_Y f(M_Y)\right)^{-2} + \left(\frac{1}{n}-\frac{1}{N}\right)\Bigg\{\eta_3^2\left(M_Z f(M_Z)\right)^{-2}$$
$$-2\eta_3\rho_{yz}\frac{\left(M_Z f(M_Z)\right)^{-1}}{\left(M_Y f(M_Y)\right)}\Bigg\} + \left(\frac{1}{m}-\frac{1}{n}\right)\Bigg\{\left(\theta^2-2\theta\right)\left(M_Y f(M_Y)\right)^{-2}$$
$$+\eta_1^2\left(M_X f(M_X)\right)^{-2} + \eta_2^2\left(M_Z f(M_Z)\right)^{-2} + 2\eta_1(1-\theta)\rho_{xy}\frac{\left(M_X f(M_X)\right)^{-1}}{\left(M_Y f(M_Y)\right)}$$
$$+2\eta_2(1-\theta)\rho_{yz}\frac{\left(M_Z f(M_Z)\right)^{-1}}{\left(M_Y f(M_Y)\right)} + 2\eta_1\eta_2\rho_{xz}\frac{\left(M_X f(M_X)\right)^{-1}}{\left(M_Z f(M_Z)\right)}\Bigg\}\Bigg] \tag{2.3}$$

For any fixed value of θ, $MSE\left(\hat{M}_{YH\theta}\right)$ is minimized for

$$\eta_1 = (1-\theta)\left(\frac{\rho_{yz}\rho_{xz}-\rho_{xy}}{1-\rho_{xz}^2}\right)\frac{M_X f(M_X)}{M_Y f(M_Y)}$$

$$\eta_2 = (1-\theta)\left(\frac{\rho_{xy}\rho_{xz}-\rho_{yz}}{1-\rho_{xz}^2}\right)\frac{M_Z f(M_Z)}{M_Y f(M_Y)}$$

$$\eta_3 = \rho_{yz}\frac{M_Z f(M_Z)}{M_Y f(M_Y)}$$

And its minimum value is given by

$$MSE(\hat{M}_{YH\theta}) = \frac{1}{4\left(f(M_Y)\right)^2}\left[\left(\frac{1}{m}-\frac{1}{N}\right)-\left(\frac{1}{n}-\frac{1}{N}\right)\rho_{yz}^2-\left(\frac{1}{m}-\frac{1}{n}\right)\left\{(1-\theta)^2 R_{y.xz}^2-\left(\theta^2-2\theta\right)\right\}\right] \tag{2.4}$$

And the bias of its optimum estimator obtained is

$$\begin{aligned} Bias(\hat{M}_{YH\theta}) = \frac{1}{8M_Y\left(f(M_Y)\right)^2\left(1-\rho_{xz}^2\right)^2}&\Bigg[\left(\frac{1}{m}-\frac{1}{n}\right)(1-\theta)\Big\{(1-\theta)\Big[\left(\rho_{yz}\rho_{xz}-\rho_{xy}\right)^2 \\ &+2\rho_{xy}\left(\rho_{yz}\rho_{xz}-\rho_{xy}\right)\left(1-\rho_{xz}^2\right)+2\rho_{yz}\left(\rho_{xy}\rho_{xz}-\rho_{yz}\right)\left(1-\rho_{xz}^2\right) \\ &+2\rho_{xz}\left(\rho_{xy}\rho_{xz}-\rho_{yz}\right)\left(\rho_{yz}\rho_{xz}-\rho_{xy}\right)+\left(\rho_{xy}\rho_{xz}-\rho_{yz}\right)^2\Big] \\ &-\left(\rho_{yz}\rho_{xz}-\rho_{xy}\right)\left(1-\rho_{xz}^2\right)\frac{M_Y f(M_Y)}{M_X f(M_X)}-\left(\rho_{xy}\rho_{xz}-\rho_{yz}\right)\left(1-\rho_{xz}^2\right)\frac{M_Y f(M_Y)}{M_Z f(M_Z)}\Big\}\Bigg] \\ &+\left(\frac{1}{n}-\frac{1}{N}\right)\rho_{yz}\left\{\frac{M_Y f(M_Y)}{M_Z f(M_Z)}-\rho_{yz}\right\}\Bigg] \end{aligned} \tag{2.5}$$

3. COMPARISON

Since the mean squared errors of optimum estimators defined by Singh et al (2006), Gupta et al (2008) and Jhajj et al (2014) is same up to first order of approximation, so we compare the proposed family of estimators with Singh et al (2006) estimators.

Using the expressions (1.2) and (2.4), we have

$$MSE\left(\hat{M}_S\right)_{\min} - MSE\left(\hat{M}_{YH\theta}\right) = \frac{1}{4\left(f(M_Y)\right)^2}\left(\frac{1}{m}-\frac{1}{n}\right)\left(\theta^2-2\theta\right)\left(R_{y.xz}^2-1\right)$$

$$> 0 \qquad for \quad 0 < \theta < 2 \tag{3.1}$$

From (3.1), we note that the proposed family of estimators is always better than Singh et al (2006) estimator, Gupta et al (2008) estimator and Jhajj et al estimator (2014) for $0 < \theta < 2$.

From the expressions of the biases obtained, we noted that no concrete interpretation can be given theoretically about the superiority of one estimator over the other on the basis of their biases because of the complicated expressions obtained in comparisons. To have a rough idea, we compare the biases numerically and graphically by taking data of populations considered in literature in Section 4.

4. NUMERICAL ILLUSTRATION

For illustration of results numerically and graphically, we take the following data

Data 1 (Source: Singh, 2003). Y: the number of fish caught by marine recreational fishermen in 1995; X: The number of fish caught by marine recreational fishermen in 1994; Z: The number of fish caught by marine recreational fishermen in 1993.

N = 69, $\rho_{xy} = 0.1505$, $M_Y = 2068$, $f(M_Y) = 0.00014$

n = 24, $\rho_{yz} = 0.3166$, $M_X = 2011$, $f(M_X) = 0.00014$

m =17, $\rho_{xz} = 0.1431$, $M_Z = 2307$, $f(M_Z) = 0.00013$

Data 2 (Source: Aezel and Sounderpandian, 2004). Y: The U.S. exports to Singapore in billions of Singapore dollars; X: The money supply figures in billions of Singapore dollars; Z: The local supply in U.S. dollors.

N = 67, $\rho_{xy} = 0.6624$, $M_Y = 4.8$, $f(M_Y) = 0.0763$

n = 23, $\rho_{yz} = 0.8624$, $M_X = 7.0$, $f(M_X) = 0.0526$

m =15, $\rho_{xz} = 0.7592$, $M_Z = 151$, $f(M_Z) = 0.0024$

Data 3 (Source: MFA, 2004). Y: District-wise tomato production (tones) in 2003; X: District-wise tomato production (tones) in 2002; Z: District-wise tomato production (tones) in 2001.

N = 97, $\rho_{xy} = 0.2096$, $M_Y = 1242$, $f(M_Y) = 0.00021$

n = 46, $\rho_{yz} = 0.1233$, $M_X = 1233$, $f(M_X) = 0.00022$

m =33, $\rho_{xz} = 0.1496$, $M_Z = 1207$, $f(M_Z) = 0.00023$

For Data 1

Table 4.1: Bias and Relative Efficiency of Estimators

	$\lvert Bias \rvert$				**Relative Efficiency**	
	$\hat{M}_S$	$\hat{M}_P$	$\hat{M}_{YH}$	$\hat{M}_{YH\theta}$	$\hat{M}_S$	$\hat{M}_{YH\theta}$
-0.13	58.62	20.22	32.57	16.28029	100	90.34
-0.08	58.62	20.22	32.57	16.64022	100	93.99
-0.03	58.62	20.22	32.57	16.97063	100	97.72
0	58.62	20.22	32.57	17.15472	100	100
0.3	58.62	20.22	32.57	18.41137	100	124.36
0.6	58.62	20.22	32.57	18.60579	100	147.62
0.9	58.62	20.22	32.57	17.73798	100	161.34
1.2	58.62	20.22	32.57	15.80795	100	158.39

Table 4.1: Contd.,						
1.5	58.62	20.22	32.57	12.8157	100	140.45
1.8	58.62	20.22	32.57	8.76122	100	116.04
2.1	58.62	20.22	32.57	3.644517	100	92.54

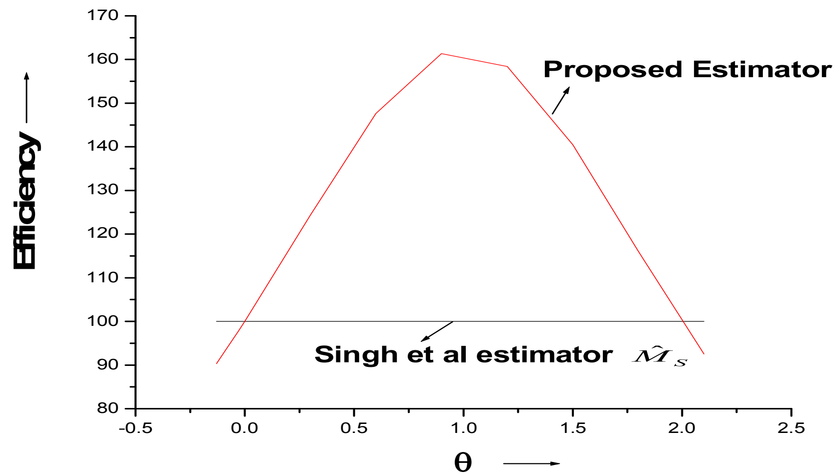

Figure 4.1

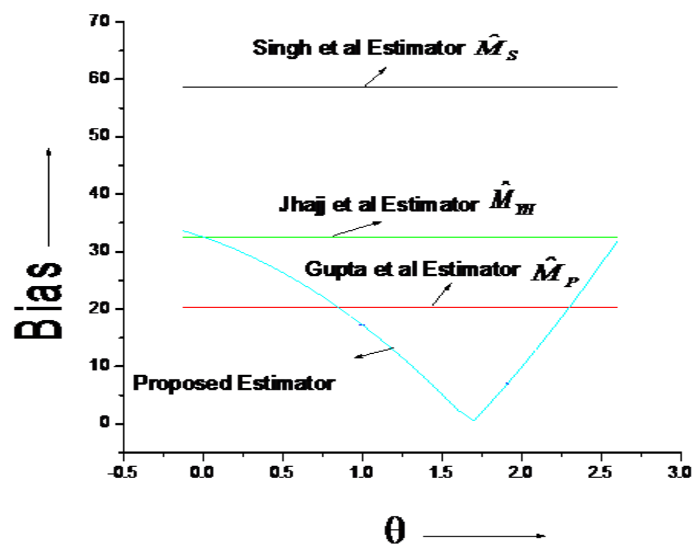

Figure 4.2

For Data 2

Table 4.2: Bias and Relative Efficiency of Estimators

θ	$\lvert Bias \rvert$				**Relative Efficiency**	
	$\hat{M}_S$	$\hat{M}_P$	$\hat{M}_{YH}$	$\hat{M}_{YH\theta}$	$\hat{M}_S$	$\hat{M}_{YH\theta}$
-1.7	0.37	0.28	0.026	0.30093	100	26.19112
-1.2	0.37	0.28	0.026	0.15731	100	36.759
-0.7	0.37	0.28	0.026	0.05226	100	54.14859
-0.2	0.37	0.28	0.026	0.014208	100	83.533
0	0.37	0.28	0.026	0.029993	100	100

Table 4.2: Contd.,						
0.4	0.37	0.28	0.026	0.043043	100	140.2007
0.8	0.37	0.28	0.026	0.031403	100	175.4711
1.2	0.37	0.28	0.026	0.00493	100	175.4711
1.6	0.37	0.28	0.026	0.06595	100	140.2007
2	0.37	0.28	0.026	0.15166	100	100
2.4	0.37	0.28	0.026	0.26207	100	69.92491
2.8	0.37	0.28	0.026	0.39716	100	49.91067
3.2	0.37	0.28	0.026	0.55695	100	36.759
3.6	0.37	0.28	0.026	0.30093	100	27.92802
4	0.37	0.28	0.026	0.15731	100	21.814

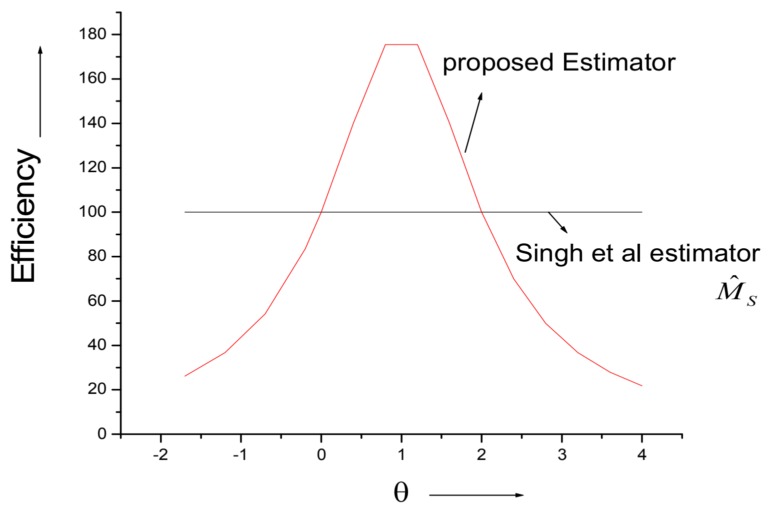

Figure 4.3

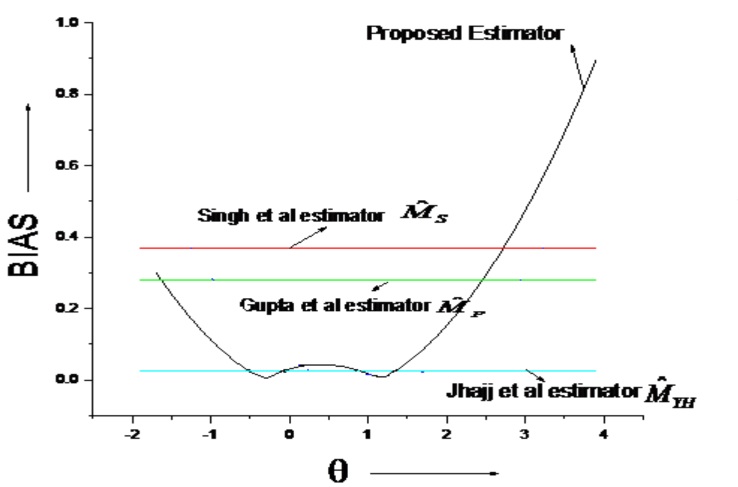

Figure 4.4

For Data 3

Table 4.3: Bias and Relative Efficiency of Estimators

θ	$\lvert Bias \rvert$				Relative Efficiency	
	$\hat{M}_S$	$\hat{M}_P$	$\hat{M}_{YH}$	$\hat{M}_{YH\theta}$	$\hat{M}_S$	$\hat{M}_{YH\theta}$
-0.1	8.06	3.63	4.75	7.323281	100	91.91
0	8.06	3.63	4.75	6.998945	100	100
0.2	8.06	3.63	4.75	6.288614	100	117.75
0.4	8.06	3.63	4.75	5.496073	100	136.63
0.6	8.06	3.63	4.75	4.621322	100	154.29
0.8	8.06	3.63	4.75	3.66436	100	167.26
1	8.06	3.63	4.75	2.625188	100	172.08
1.2	8.06	3.63	4.75	1.503806	100	167.26
1.4	8.06	3.63	4.75	0.300213	100	154.29
1.6	8.06	3.63	4.75	0.98559	100	136.63
1.8	8.06	3.63	4.75	2.3536	100	117.76
2	8.06	3.63	4.75	3.80383	100	100
2.2	8.06	3.63	4.75	5.33626	100	84.44

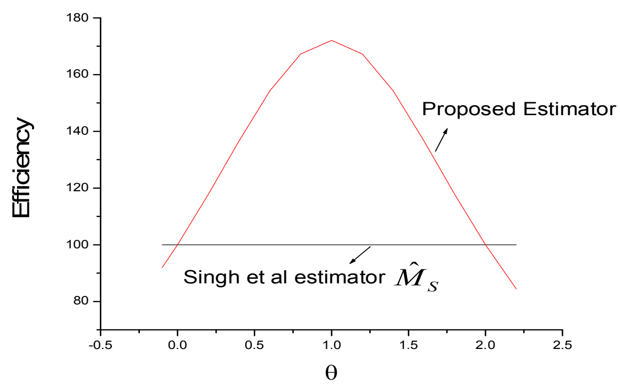

Figure 4.5

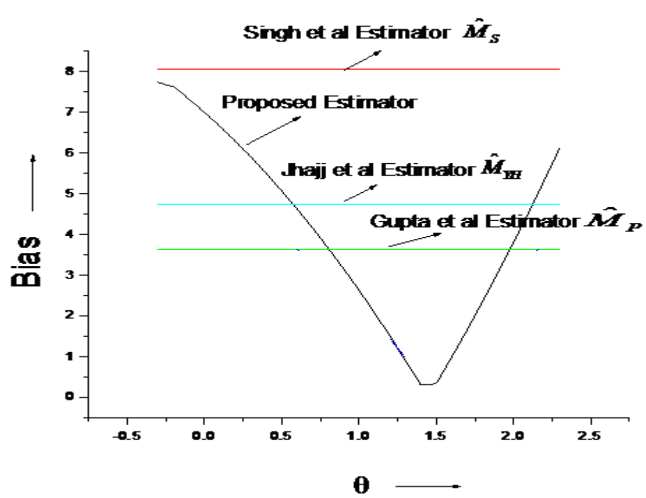

Figure 4.6

From tables (4.1), (4.2) and (4.3), we noted that proposed family of estimators have smaller mean squared error than Singh et al (2006) estimator, Gupta et al (2008) estimator and Jhajj et al estimator (2014) as well as have smaller bias than Singh et al (2006) estimator, Gupta et al (2008) estimator and Jhajj et al estimator (2014) in all the three populations considered corresponding to $\theta \in (0,2)$. The graphical representation also supports the numerical results.

CONCLUSIONS

It is interesting to note that for any value of $\theta \in (0,2)$, the proposed sampling strategy for estimating the population median is efficient than the sampling strategies obtained by Singh et al (2006) estimator, Gupta et al (2008) estimator and Jhajj et al estimator (2014). Numerical results obtained corresponding to all populations considered in literature also justify that proposed sampling strategy of estimating population median having smaller bias is efficient than existing ones. Comparison of the estimators is shown graphically based upon the numerical results obtained also give same type of trend. Therefore, we strongly recommended that proposed sampling strategy should be used for getting the accurate value of population median.

REFERENCES

1. *Azel A.D., Sounderpandian J. (2004).Complete Business Statistics. 5th ed. New York: McGraw Hill,*
2. *Gross S.T. (1980). Median estimation in sample Proc. Surv. Res. Meth. Sect. Amer. Statist Ass.,181-184.*
3. *Gupta S., Shabbir J. and Ahmad S.(2008). Estimation of median in two- phase sampling using two auxiliary variables. Communication in Statistics-Theory and Methods, 37(11), 1815- 1822.*
4. *Jhajj H.S.and Harpreet Kaur (2013),Generalized estimators of population median using auxiliary information, IJSER, 11(4), 378-389.*
5. *Jhajj H.S., Harpreet Kaur and Walia Gurjeet(2013), Efficient family of ratio-product type estimators of median, MASA, Vol 9, 2014, 277-282.*
6. *Jhajj H.S., Harpreet Kaur and Jhajj Puneet (2014), Efficient family of estimators of median using two phase sampling design, Communication in Statistics-Theory and Methods, Accepted.*
7. *Kuk Y.C.A. and Mak T.K. (1989). Median estimation in the presence of auxiliary information. J.R Statist. Soc. B, (2), 261-269.*
8. *MFA (2004). Crops Area Production. Ministery of food and Agriculture. Islamabad: Pakistan.*
9. *Singh S. (2003).Advanced Sampling Theory with Applications. Vol. 1 & 2. London Kluwer Academic Publishers.*
10. *Singh S., Joarder A. H. and Tracy D. S.(2001), Median estimation using double sampling. Austral. N.J. Statist. 43(1), 33-46.*
11. *Singh S., Singh H.P and Upadhyaya L.N. (2006). Chain ratio and regression type estimators for median estimation in survey sampling. Statist. Pap. 48, 23-46.*